Exercises

1. Prove the Euler Identities of Lecture 1.
2. Given a mixed polynomial $f : (\mathbb{C}^n, 0) \to (\mathbb{C}, 0)$, describe the Milnor set $M(f)$.
   (For definition of Milnor set, please look up Course D-Part 3, by Prof. Mihai Tibăr)
3. Show that the following Oka's example is non-degenerate but not strongly non-degenerate,
   $$f = \frac{1}{4}(z_1^2 - \bar{z}_1^2) + \|z_1\|^2 - (1+i)\|z_1 + z_2\|^2.$$
4. In holomorphic case, Kouchnirenko proved that the non-degeneracy condition is a generic condition(In the sense of Zariski's topology). Please give an example to explain that the strong non-degeneracy condition for mixed polynomials is neither dense nor connected.
5. For a polar weighted homogenous mixed polynomial $f : (\mathbb{C}^n, 0) \to (\mathbb{C}, 0)$ not necessary to be strongly non-degenerate, does it always have a Milnor tube fibration?
6. Let $V(f) = \{z_1^2\bar{z}_1 - 2z_2^2 = 0\}$. Give a good resolution for the singularity of $V(f)$.
7. Assume that $f : (\mathbb{C}^2, 0) \to (\mathbb{C}, 0)$ is
   $$f = (z_1^3\bar{z}_1^2 - 2z_2^2)(z_1^4\bar{z}_1^3 - 3z_2^3\bar{z}_2).$$
   Please find $\mathrm{lkn}(f^{-1}(0), 0)$, $\mu(f)$ and the zeta function of the monodromy map.
8. For any $k \in \mathbb{N}$, construct a mixed polynomial $f : (\mathbb{C}^2, 0) \to (\mathbb{C}, 0)$ such that the Milnor number $\mu(f) = k$.