

SELECTED BIBLIOGRAPHY FOR CIMPA 2021 TALKS

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1. SEVERAL BOOKS WITH MATHEMATICAL APPROACHES FOR IMAGING

Included in the bibliography is a short selection of books which describe various methods for the analysis of computer images. Several include the methods which make specific use of singularity theory, referred to in the lectures, e.g. [DGH], [PS], [Pe]. In addition, there are several which cover methods which were indirectly referred to in the lectures, e.g. [Ko], [Yo],[MS]. In addition, there are several others also describing other methods, which for time considerations, were not mentioned, e.g. [SNFJ], [Mal], [Se]. These books also contain references to many papers on these topics. There are too many other books to mention on various topics in computer imaging.

2. REFERENCES FOR THE BLUM MEDIAL AXIS AND SKELETAL STRUCTURES

These papers develop the introduction and generic structure of Blum medial axis for regions in \mathbb{R}^2 and \mathbb{R}^3 , [BN], [Y], [BGG], [GK], [GK2] and more generally \mathbb{R}^n [M]. Also included are references for the introduction of skeletal structures in \mathbb{R}^n and the derivation of the mathematics of “medial geometry” applied to both local and global geometry, and the geometric complexity of regions in \mathbb{R}^3 [D1] - [D6]. There are many additional papers developing various aspects of the Blum medial axis and its explicit computation, e.g. [SBTZ]. Many additional references can be found in the book [PS].

3. REFERENCES FOR THE MEDIAL/SKELETAL LINKING STRUCTURES

The introduction and development of the generic properties of Blum medial linking structures and the generalization to skeletal linking structures, with the derivation of their resulting mathematical properties, are contained in [DG], [DG1], [DG2], [Ga].

4. SELECTED PAPERS ON MEDICAL IMAGING

Listed below is just a very brief selection of papers on the applications of the uses of medial or skeletal type structures as deformable templates for medical imaging, developed by the MIDAG group at UNC. Still many more are listed in the book [PS].

5. SELECTED PAPERS FOR NATURAL IMAGES

We include references to papers where singularity theory is used for determining the generic local structures of natural images and the changes under generic transitions under viewer movement, [DGH1], [DGH2], and the book [DGH]. As well we have included references to results on natural images mentioned in the lectures. There are many more references included in the book [DGH].

6. RELEVANT REFERENCES FOR SINGULARITY THEORY USED FOR IMAGING PROBLEMS

The two main applications of singularity theory involve Thom-Mather theory and stratifications.

References for Thom-Mather Theory.

The key references for the smooth part of Thom-Mather Theory are Mather's papers [MI]- [MIV], where he considers the results for the groups \mathcal{A} , \mathcal{K} , \mathcal{R} , and \mathcal{L} . There are standard references in a number of books which cover various aspects for this theory, see e. g. [GG], [AGV] and [Mt]. A more elementary introduction covering \mathcal{R} -equivalence, with some relevant applications to differential geometry is given in [BG1], and a very recent book covering Thom-Mather Theory for these groups in both the smooth and complex analytic cases is [MNB].

The part of the theory used for the generic projections from smooth surfaces in \mathbb{R}^3 to a plane is contained in [Wh], [M2]; and the mathematical classifications up to \mathcal{A}_e -codimension 2 are obtained in [Gaf] and [A1].

For the more general version of Thom-Mather theory applicable to more general groups of equivalence, [D7a] describes a general form which applies, with a large number of examples, as does the first few sections of [D10], and the detailed proofs are given in [D7b]. The topological versions for the general Thom-Mather Theory used here differ from the stratification methods developed by Thom and Mather for topological stability of mappings, but instead, use infinitesimal methods and are given in [D8a], with a number of examples in [D8b]. Classification results in this framework applicable to projections for natural images are contained in [BG2], [Go], and [Ta] as well as the book [DGH], where the methods given in [BDW], [BKD], [Kr], and the software "Transversal" for Maple 5 were extensively used in the classifications.

References for Stratifications.

There are many different directions in which stratification theory has developed. We concentrate on those relevant for the imaging questions. Whitney stratifications were introduced in [Wh2], and further properties for their use in topological stability can be found in [M3], [M4], and [GWDL]. Whitney stratifications occur generically for the Blum medial axis by [M], and are used in the definition of skeletal structures [D1].

For applying singularity theory to natural images, an additional algebraic structure on the stratifications is required, in the form of "special semi-analytic stratifications" see [DGH]; and this has a generalization to "semi-coherent stratifications", for which the general Thom-Mather theory still applies, see [D9].

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Selection of Books on Computer Imaging

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