

Program of Lectures

Cimpa Research School - Singularities and Applications

11–22, July 2022.

FIRST WEEK: 11–15, July 2022

Course 1. Singularities and algebraic methods. Lecturers: Maria Ruas, Juan Nuño Ballesteros, Bruna Oréface, Roberta Wik Atique, Terence Gaffney, Matthias Zach, Jim Damon.

Course 1A (C1A). Complete intersections with isolated singularities (ICIS).

Course 2 (C2). Algorithmic and computational aspects of singularities. Lecturers: Anne Frühbis-Krüger, Matthias Zach, Sabrina.

Follow up sections

F2. Milnor fibrations, nonisolated singularity, and the polar degree of projective hypersurfaces. Lecturers: Mihai Tibăr, Dirk Siersma.

F3. Topology of complex projective hypersurfaces and of their complements. Lecturer: Laurentiu Maxim.

F4. Real Milnor fibrations, and mixed singularities. Lecturers: Ying Chen, Raimundo Araújo dos Santos, Mihai Tibăr, Nicolas Dutertre.

F5. Bifurcation of affine maps in real and complex settings. Lecturers: Luis Renato Dias, Cezar Joita, Mihai Tibăr.

TIMETABLE FIRST WEEK

Day Time	Monday 11th	Tuesday 12th	Wednesday 13th	Thursday 14th	Friday 15th
08h–08:30	Opening	–	–	–	–
08:30–09:15	C1A(Bruna+Roberta)	C2	C1A(Roberta)	C1A(Juan)	C1A(Terry)
09:20–10:05	C1A(Bruna)	C2	C1A(Cidinha)	C1A(Juan)	F3(Max)
10:10–10:40	Break	Break	Break	Break	Break
10:40–11:25	F2(Mihai)	F3(Max)	C2	F4(Mihai)	Posters
11:30–12:15	F2(Dirk)	F3(Max)	—	F5(Renato)	Posters
12:20–14:30	Lunch	Lunch	Lunch	Lunch	Lunch
14:30–15:15	F5(Cezar)	F4(Dirk)	Free Disc.	C2	C1A(Terry)
15:20–16:05	F5(Cezar)	F5(Renato)	Free Disc.	C2	F3(Max)
16h10–16:40	Break	Break	Free Disc.	Break	Break
16:40–17:25	Exer(Dirk)	Exer(Max)	Free Disc.	Exer(C1A)	Exer(Dirk)
17:30–18:15	Exer(Cezar)	Exer(Zach)	Free Disc.	Exer(C1A)	Free Disc.

Program of Lectures

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SECOND WEEK: 18–22, July 2022

Course 1. Singularities and algebraic methods. Lecturers: Maria Ruas, Juan Nuño Ballesteros, Bruna Oréface, Roberta Wik Atique, Terence Gaffney, Matthias Zach, Jim Damon.

Course 1B (C1B). Determinantal Singularities.

Course 2 (C2). Algorithmic and computational aspects of singularities. Lecturers: Anne Frühbis-Krüger, Matthias Zach, Sabrina.

Course 3 (C3). Lipschitz geometry of singularities. Lecturers: Alexandre César, Edson Sampaio, Mihai Tibăr, Lev Birbrair, Zbigniew Jelonek.

Follow up sections

F2. Milnor fibrations, nonisolated singularity, and the polar degree of projective hypersurfaces. Lecturers: Mihai Tibăr, Dirk Siersma.

F3. Topology of complex projective hypersurfaces and of their complements. Lecturer: Laurentiu Maxim.

F4. Real Milnor fibrations, and mixed singularities. Lecturers: Ying Chen, Raimundo Araújo dos Santos, Mihai Tibăr, Nicolas Dutertre.

F5. Bifurcation of affine maps in real and complex settings. Lecturers: Luis Renato Dias, Cezar Joita, Mihai Tibăr.

Other notations

TIMETABLE SECOND WEEK

Day Time	Monday 18th	Tuesday 19th	Wednesday 20th	Thursday 21st	Friday 22nd
08:30–09:15	C1B(Cidinha)	C1B(Cidinha)	C2	C3(Edson)	C3(Zbigniew)
09:20–10:05	F2(Dirk)	C2	C2	C3(Alexandre)	C3(Lev)
10:10–10:40	Break	Break	Break	Break	Break
10:40–11:25	C3(Alexandre)	C3(Edson)	C1B(Terry)	C1B(Jim)	C1B(Zach)
11:30–12:15	C3(Alexandre)	C3(Edson)	F4(Nicolas)	C1B(Zach)	C1B(Jim)
12:20–14:30	Lunch	Lunch	Lunch	Lunch	Lunch
14:30–15:15	C2	F2(Dirk)	Junior Talks	Exer(C1B)	
15:20–16:05	F4(Nicolas)	C1B(Terry)	Junior Talks	Exer(C1B)	
16:10–16:40	Break	Break	Junior Talks	Posters	
16:40–17:25	Exer(Nicolas)	Exer(Zach)	Junior Talks	Posters	
17:30–18h15	Exer(Terry)	Exer(Alex)	Junior Talks		

Junior Scientific Activities

We will offer two junior scientific activities consisting of a “**Junior Talks session**” and “**Poster presentation session**”. Junior Talks consists of 30-minute lectures given by PhD students and post-docs researchers with the aim of showing the recent progress in their projects. The posters are an opportunity to masters and Ph.D. students to show their ongoing scientific projects through posters presentations. More details can be found below.

1. Posters section

- (1) Aline Bartel, Carl von Ossietzky Universität Oldenburg, Germany.
- (2) Maicom Douglas Varella Costa, Carl von Ossietzky Universität Oldenburg, Germany.
- (3) Reinaldo Resende de Oliveira, IME-USP, Brazil.

2. Junior Speakers section

- (1) Alexander Otto **Hop**, Wisconsin University, USA.
- (2) Bárbara Karolline de Lima **Pereira**, UFSCar, Brazil.
- (3) Eder **Sanchez**, ICMC-USP, Brazil.
- (4) Gabriel **Monsalve**, Université de Lille, France.
- (5) Hellen **Santana**, ICMC-USP, Brazil.
- (6) Yovani Adolfo Villanueva **Herrera**, UFG, Brazil.

Timetable Junior Talks	
Day	Wednesday 20th
14:30–15:00	A. Hop
15:00–15:30	B. Pereira
15:30–16:00	E. Sanchez
16:00–16:30	Break
16:30–17:00	G. Monsalve
17:00–17:30	H. Santana
17:30–18:00	Y. Herrera

Scientific Committee

Anne Frühbis-Krüger, Universität Oldenburg, Germany

Maria Aparecida Ruas, ICMC-USP, Brazil

Mihai Tibăr, Université de Lille, France

Raimundo Araújo dos Santos, ICMC-USP, Brazil

Titles and Abstracts of the Talks

Milnor Fiber Consistency for Deformations of Arbitrarily-Singular Hypersurfaces

Alexander Otto Hof
University of Wisconsin-Madison, US.

Abstract: The Milnor fiber of a holomorphic function defining an isolated singularity can be understood by perturbing the function slightly to one with only Morse critical points. For an arbitrary-dimensional singularity, perturbation is no longer guaranteed to preserve the Milnor fiber, making an analogous approach difficult. We present a new algebraic condition - the flatness of the critical locus over the parameter space - under which this problem does not arise, giving a new avenue for Milnor fiber computations.

The Bruce Roberts Numbers of a Function on an Isolated Complete Intersection Singularity

Bárbara Karolline de Lima Pereira
Federal University of Sao Carlos, Brazil.

Abstract: In this talk I will give formulas for the Bruce-Roberts number $\mu_{BR}(f, X)$ and its relative version $\mu_{BR}^-(f, X)$ of a function f with respect to an ICIS $(X, 0)$. We show that $\mu_{BR}^-(f, X) = \mu(f^{-1}(0) \cap X, 0) + \mu(X, 0) - \tau(X, 0)$, where μ and τ are the Milnor and Tjurina numbers, respectively, of the ICIS. The formula for $\mu_{BR}(f, X)$ is more complicated and also involves $\mu(f)$ and some lengths in terms of the ideals I_X and Jf .

Real algebraic links associated to mixed singularities

Eder Sanchez
ICMC-USP, Brazil.

Abstract: We will show how to construct new classes of mixed singularities that provide realizations of real algebraic links in the 3-sphere. These new classes of mixed singularities may help to elucidate the Benedetti-Shiota conjecture which states that any fibered link on the 3-sphere is a real algebraic link.

(*) The research has been supported by São Paulo Research Foundation (FAPESP), grants 2017/25902-8 and 2019/11415-3.

**Title Milnor set of real polynomials
and detection of atypical values**

Gabriel Monsalve
Université de Lille, France.

Abstract: Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ be a polynomial function. The Milnor set at infinity of f is defined as the set of points where the fibers of f are tangent to large enough spheres in \mathbb{R}^2 . In this talk we present the importance of such a set to characterize the regular atypical values of f . Our technique is based in the detection of phenomena known as vanishing and splitting (of the fibers) at infinity, using special branches of the Milnor set called μ -clusters.

(*) We would like to thanks BEPE-grant #2020/14111-2 São Paulo Research Foundation (FAPESP) by financial support.

**About the local topology of function-germs
with a one-dimensional critical set**

Hellen Santana
ICMC-USP, Brazil.

Abstract: In this work, we consider two function-germs $f, g : (X, 0) \rightarrow (\mathbb{C}, 0)$ such that f has a stratified isolated singularity at the origin and g has a stratified one-dimensional critical set. We study the local topology of a deformation \tilde{g} of g defined by $\tilde{g} = g + f^N$, where $N \gg 1$ and $N \in \mathbb{N}$ and we present the relation between the critical sets of g and \tilde{g} .

Normal Forms of Constrained Differential Systems

Yovani Adolfo Villanueva Herrera
Federal University of Goias, Brazil.

Abstract: The subject of this work is the theory of normal forms of smooth vector fields of constrained systems (systems of non-linear differential-algebraic equations). In this study we introduce the qualitative theory of ordinary differential equations, with topics such as stability, structural stability, bifurcations, limit cycles and catastrophes of differential equations, and the functional singularity theory. The goal of this work is classify and normalize constrained systems, first of all from the local point of view, we'll show an idea of the global one based on piecewise differential systems and our final objective will be to consider this theory to differentiable manifolds of dimension $n \geq 2$.